

## On Mesh Smoothing and Material Advection for Modelling Pile Installation in Sand

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Pile installation in sand is one of the most complex problems in soil mechanics, and the numerical modelling of pile installation, i.e. the penetration of soil, is still an important topic of research. The main difficulties arise from the very large deformations and deep indentation of the soil material, the changing contact states, the complex mechanical behavior of sand, and the multiple length and stiffness scales. The classical Lagrangian and Eulerian versions of the finite element method are often not suitable to model problems involving large material deformations. In the total and updated Lagrangian approaches, the element mesh is stuck to the material and follows the material deformations. Therefore, solution may fail to proceed due to severe element distortion just after a small amount of penetration has occurred. In this case rezoning of the material domain, which consists of remeshing plus projection of the solution, becomes unavoidable. However, frequent rezoning is computationally expensive, because the connectivity of every new generated mesh has to be redetermined. Moreover, simple projection methods based on interpolation introduce errors, as they are not conservative. On the other hand, Eulerian finite element approaches keep the mesh fixed in space, but the treatment of path dependent constitutive equations, free surfaces, and moving boundaries becomes hard to challenge.

The arbitrary Lagrangian-Eulerian (ALE) formulation [1, 2] has been developed to overcome the difficulties arising from the Lagrangian and Eulerian viewpoints, and to combine their advantages. From a geometric point of view, the ALE finite element mesh represents an arbitrary moving reference manifold that is diffeomorphic to the initial configuration and the current configuration of the material body [3, 4]. The physical motion of the body is then understood to be a composition of two maps involving the reference domain. Since that early days the ALE framework has grown up to a powerful analysis tool for large deformation problems, but applications in computational soil mechanics are rare; for a recent example, see [5]. ALE finite element approaches avoid complete remeshing. Instead, the mesh is smoothed so that element distortion is reduced while mesh topology is kept unchanged. As an element has the same neighbors during the whole calculation, conservative advection algorithms developed for fluid dynamical problems can be applied to project the solution onto the modified mesh.

Along with a research project, we are currently implementing an ALE method into a commercial finite element code in order to simulate pile installation in sand numerically. The mechanical behavior of sand is complicated, because it depends on the stress state and stress history, as well as on the density state. This means that density changes must explicitly be accounted for to accomplish conservation of mass with respect to a moving mesh. As it is to be shown in the presentation, our ALE method applies the operator-split according to [6], that facilitates the incorporation of a complex hypoplastic constitutive equation for sand [7] in a pure Lagrangian step. The Lagrangian step is followed by a mesh smoothing step. Mesh smoothing plays a crucial role in the simulation of pile installation. However, the explicit smoothing algorithms applied in several ALE methods are inapplicable to the non-convex mesh regions around the pile tip that occur as penetration proceeds. We present an implicit optimization scheme that works quite well on structured and unstructured triangle meshes over convex and non-convex

domains. After mesh smoothing, the element Jacobian, the stress, the void ratio of the sand, and the history variables are remapped onto the modified mesh. This transport (or Eulerian) step is performed by a simple Godunov-like advection scheme [8]. Our presentation is completed with some numerical examples, including benchmark tests for the algorithms, and simulations of pile installation in sand.

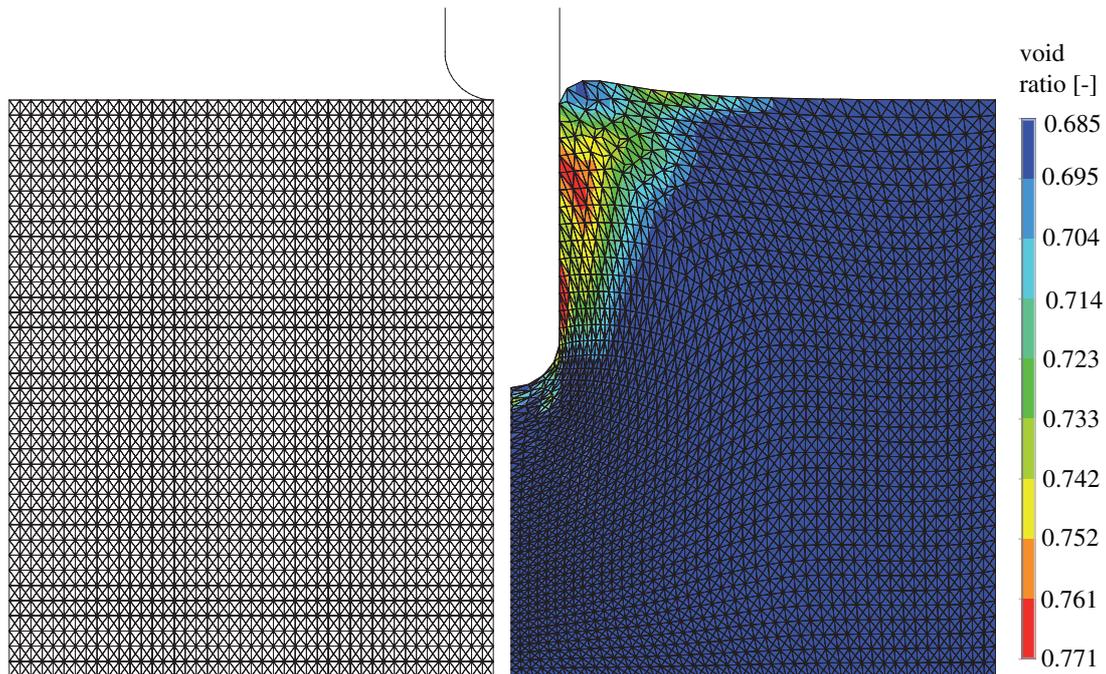


Figure 1: Pile installation in sand. *Left*: initial configuration and mesh, *right*: deformed mesh and void ratio distribution (ALE formulation).

## References

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