ABSTRACT: The performance of two advanced numerical methods using Multi-Material Arbitrary Lagrangian-Eulerian (MMALE) and Coupled Eulerian-Lagrangian (CEL) formulations is studied. The evaluation is based on two large deformation benchmark cases which classical pure Lagrangian methods cannot model. MMALE is an enhanced version of CEL in which the computational mesh can be rezoned in an arbitrary way so that mesh nodes are concentrated in areas of interest. This form of solution adaptivity provides more data in regions undergoing large deformations compared to the fixed mesh in CEL methods. MMALE has gained popularity in the field of fluid dynamics. In this study, the applicability of MMALE to geomechanical problems is investigated with regard to accuracy and robustness. Two geomechanical problems, pipeline displacement and sand column collapse, are analyzed for this purpose. It can be concluded that MMALE handles such large deformation problems more efficiently than CEL.

1 INTRODUCTION

In computational geomechanics, the numerical simulation of soil-structure-interaction problems where the soil material undergoes large deformations has become an active area of research (Aubram et al. 2017; Wang et al. 2015). Classical finite element methods (FEM) based on a Lagrangian formulation suffer from mesh elements distortion which may deteriorate accuracy or even stop the solution at early stages of the calculation. Novel techniques and advanced numerical methods have been developed to resolve these issues associated with the Lagrangian approach. These methods have been proven a powerful and reasonably accurate alternative to experimental and analytical solutions. The Arbitrary Lagrangian-Eulerian (ALE) and the Coupled Eulerian-Lagrangian (CEL) are two of such methods.

CEL has been extensively evaluated in the context of geotechnical problems. For example, a study done by (Wang et al. 2015) compared the performance of CEL with other numerical methods for large deformation geotechnical problems. Another study thoroughly evaluated the CEL compatibility with a complex soil material model and the results were compared to an experimental test (Bakroon et al. 2017a).

On the other hand, application of ALE to geotechnical problems is rather new and mostly uses a simplified mesh formulation (so-called simplified or single-material ALE). Moreover, current ALE research is focused on rather technical aspects of the method (Barlow et al. 2016; Aubram 2013; Aubram et al. 2017), hence further studies are required concerned with its application in geotechnical engineering. A recent work done by (Bakroon et al. 2017b) compares simplified ALE with the classical FEM using a benchmark case where an analytical solution is available. They conclude that ALE provides more accurate and stable results when applied to large deformation geotechnical problems. A more advanced variant of ALE, called Multi-Material ALE (MMALE), is studied in this paper, and two example applications are used to thoroughly evaluate its performance.

2 METHODS

Both ALE and CEL calculations are based on the operator-split scheme (Benson 1992); see Figure 1. First, a Lagrangian step is performed, where the computational mesh deforms with soil particles. In the second step, a new mesh is generated which is called the remeshing resp. rezoning step. The difference of the CEL method and the ALE method arises in this step (Figure 2). In CEL, the rezoned mesh is simply the original computational mesh while in ALE a new distinct mesh is generated. Finally, the solution from the old mesh is transferred into the new mesh which is called the remapping/advection step. This step is comparable to the solution in a classical Eulerian method.

ALE methods can be subdivided into two main groups, single-material/simplified ALE (SALE) and Multi-Material ALE (MMALE). The SALE method handles one material per element while MMALE can consider multiple materials in one single element. The MMALE is considered more powerful since the performance of SALE is strongly influenced by material interfaces. In other words, the remeshing step is
constrained by material boundaries, which is not the case in MMALE chosen here.

In this study, the MMALE method is evaluated against CEL method via two benchmark problems: pipeline displacement and sand column collapse. Each problem tackles specific aspects of the numerical approaches. The problems are checked against analytical and experimental results as well.

Figure 1. Diagram of the operator-split scheme applied in CEL and MMALE methods

Figure 2. Schematic diagram of SALE, CEL, and MMALE approaches comparing the effect of mesh rezoning and advection steps of the solution (Bakroon et al. 2017b).

3 NUMERICAL EXAMPLE 1: PIPELINE DISPLACEMENT

3.1 Background

Pipelines are one of the key components in offshore industrial projects. Pipes are initially placed on the seabed. After installation, the pipe penetrates the soil due to its own weight. Moreover, the varying thermal effects of the pipe induce a lateral force resulting e.g. in a lateral movement. Calculating combined horizontal and vertical resistance of the soil against pipe movement can lead to a more optimized and safe design. There is a large amount of literature concerned with various aspects of embedded pipeline behavior in seabed in the field of theoretical, physical, and numerical modeling (Merifield et al. 2009; Wang et al. 2015).

The vertical and horizontal resistance force is usually calculated based on bearing capacity theory for a shallow embedded footing (Skempton 1951). The equations are modified to take the problem conditions into account such as soil heaving, buoyancy, shape of pipe etc.

The schematic view of the problem is illustrated in Figure 3. After a pipe with diameter $D$ penetrates to a depth $w$, the soil starts to heave to a width of $B_{\text{heave}}$ with the height of $H_{\text{heave}}$. This increases the lateral resistance of the soil which can be taken into consideration to reach an optimum design.

The analytical equations calculating the horizontal and vertical resistance forces are presented in Equations 1 and 2 (cf. (Merifield et al. 2009)). These equations consist of two terms. The first term is attributed to the undrained soil strength while the second term considers the self-weight effects of the soil.

\[
\frac{V}{D} = N_{cV} s_v + N_{swV} \gamma' w \\
\frac{H}{D} = N_{cH} s_v + N_{swH} \gamma' w
\]

\[
N_{cV} = a_w^{b} = 5.3 \tilde{w}^{0.25}
\]

\[
N_{cH} = c_w^{d} = 2.7 \tilde{w}^{0.64}
\]

\[
N_{swV} = \frac{1}{2 \tilde{w}} \left( 1 + \frac{1}{\lambda} \right) \times \left[ \sin^{-1} \left( \frac{4 \tilde{w}(1 - \tilde{w})}{2} \right) - \left( 1 - 2 \tilde{w} \right) \sqrt{\tilde{w}(1 - \tilde{w})} \right]
\]

\[
N_{swH} = \frac{\tilde{w}}{2} + \frac{4}{\lambda} \times \left[ \sin^{-1} \left( \frac{4 \tilde{w}(1 - \tilde{w})}{2} \right) - \left( 1 - 2 \tilde{w} \right) \sqrt{\tilde{w}(1 - \tilde{w})} \right]
\]

where $V$ = vertical resistance force, $H$ = horizontal resistance force; $\tilde{w}$ = normalized penetration depth ($\tilde{w} = w/D$); $\lambda$ = parameter approximating the amount of heaving; $\gamma'$ = submerged unit weight of soil; $D$ = pipe
diameter; \(N_{cv}\) and \(N_{ch}\) = coefficient for vertical and horizontal strength, respectively; and \(N_{swv}\) and \(N_{swh}\) = coefficient for vertical and horizontal strength for self-weight term, respectively (see Figure 3).

As suggested by (Merifield et al. 2009) the values of 3 and 1.6 are used for \(\lambda\) in vertical and horizontal force, respectively, as well as the corresponding coefficients \(a\), \(b\), \(c\), and \(d\) in Equations 3 and 4.

3.2 Numerical model

In this problem, a pipe is placed above the soil which represents the seabed. The soil is considered to be fully saturated with average shear strength, \(\sigma_u\) = 1.5 kPa in overall depth. The submerged unit weight of the soil is \(\gamma' = 6\) kN/m\(^3\). The Young’s modulus \(E\) of the soil is calculated by \(E/\sigma_u = 500\). The elastoplastic material model with Tresca yield criterion was employed. Due to significant strength difference between pipe and soil, the pipe was considered as a rigid part. No friction was considered between pipe and soil (smooth surface).

The pipe is moved in vertical direction until depth of \(1.0D\) to simulate the embedment of the pipe. The velocity rate of 0.01 m/sec was assigned to ensure quasi-static loading conditions. The vertical resistance force was calculated and compared to analytical equations.

To compare the horizontal resistance of the pipe with analytical solution, ten models were developed, where the pipe was displaced horizontally at different embedment depths from \(0.1D\) to \(1.0D\) with intervals of \(0.1D\). No vertical displacement was allowed at this stage.

MMALE and CEL methods are used for numerical simulation. The mesh configuration of the model is shown in Figure 4. The minimum element size was 0.04 m which increased at the boundaries to 0.15 m resulting in total number of 10,900 elements. A void layer of 1 m height was defined above the soil layer to allow soil heaving simulation. The model was considered as a 2D problem, however, 3D solid elements were used since no 2D Eulerian elements are available. The thickness in normal direction is considered as one element.

3.3 Results and discussion

A Lagrangian model was first adopted. The quality of mesh elements reduced drastically after significant penetration as shown in Figure 5. Hence, the results were considered unreliable. This emphasizes on a need for more advanced models for this problem.

Subsequently, MMALE and CEL methods were used for the simulation. Both methods converged to solution after \(1.0D\) penetration. Vertical and horizontal resistance forces of CEL and MMALE are checked against analytical equations in Figure 6 and Figure 7, respectively. The vertical resistance force for both CEL and MMALE are in a good agreement with analytical equations. In Figure 6, it is shown that both methods provide acceptable results. The CEL method gives stiffer behavior than MMALE. After about 0.2m penetration, both results from the MMALE and CEL with coarse mesh, start experiencing oscillation, while the MMALE with fine-mesh is smoother. It can be argued that due to coarser mesh, less coupling nodes area available which causes the oscillation. Nevertheless, this oscillation does not cause significant errors.
For calculated horizontal force in Figure 7, both methods give higher values at low penetration depths, but lower values at higher depths in comparison to analytical equation. This can be attributed to complex mechanism of heaving and its effect on the resistance force mentioned in (Merifield et al., 2009).

By using the same model configuration, MMALE captures a better mesh resolution for areas of interest than CEL. This argument is supported by Figure 8 where CEL and MMALE interfaces are compared together using the initial mesh element size. MMALE provides a smoother interface than CEL. Hence, it is possible to achieve an acceptable accuracy with increasing the element size in MMALE.

The velocity vectors of soil after application of vertical displacement are shown in Figure 9a and Figure 9b. The velocity vectors of the horizontal displacement after reachingodel 0.5D penetration are shown in Figure 9c and Figure 9d. The arrow at center of the pipe shows its movement direction.

The velocity field shows clearly which part of the soil regime undergoes significant movement. This movement is due to the shear band mechanism appearing due to excessive pipe movement and soil softening.

In Figure 9a and Figure 9b, the soil flow regime is distinguished by dense arrows. This is similar to failure mechanism of a strip footing in general soil mechanics theory. In Figure 9c and Figure 9d, a new shear zone is developed. At both displacement modes the velocity field is uniform which is a criterion for stability of the numerical methods.

A more realistic model has been developed to account for simultaneous displacement of pipe in horizontal and vertical direction. Similar to previous model, the pipe is initially placed above the soil. Then the pipe moves vertically with the rate of 0.01 m/s into the soil until the depth of 0.5D for simulation of partial embedment. Subsequently, the horizontal displacement was applied with the same rate of 0.01 m/s. During horizontal movement, 60% of the obtained maximum vertical force in the last phase was maintained to model the self-weight of the pipe and its containing fluid. Vertical movement is allowed during this stage.

The model was solved with both CEL and MMALE. To reduce the number of irrelevant affecting variables, the simulations were conducted under same configurations and conditions on a conventional personal computer with 4-core CPU with 3.2 GHz.

Figure 10 shows the final deformed shape for both CEL and MMALE. The mesh in MMALE model is significantly concentrated around the pipe, which is also the interested area of study. In addition, due to mesh concentration, more nodes are available which enhances coupling with Lagrangian elements leading to more accurate results. Besides, more Eulerian elements at coupling interface reduce the possibility of leakage. In Figure 11 the velocity field of the soil is shown. The arrow in the middle of the pipe shows the pipe movement direction. Compared to Figure 9, more soil volume is displaced. Results obtained from the model agrees well with similar tests available in the literature (Dutta et al. 2015).

Figure 12 shows a comparison of horizontal force for MMALE and CEL. Both results converge to a similar value with negligible differences.
Furthermore, the calculation time for both CEL and MMALE is considered as a comparison criterion. As illustrated in Table 1, MMALE was about 35% faster than CEL. Although MMALE has one more step in calculation process (e.g. remeshing step), which increases calculation cost in comparison to CEL, it is not necessary to perform it at each calculation step. Hence, the remeshing and remapping step can be performed after several Lagrangian steps without affecting the results, which leads to less computation time.

Table 1. Calculation time comparison for MMALE and CEL for pipeline displacement problem

<table>
<thead>
<tr>
<th>Numerical method</th>
<th>Calculation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEL</td>
<td>02:38:43</td>
</tr>
<tr>
<td>MMALE</td>
<td>01:42:14</td>
</tr>
</tbody>
</table>

Figure 9. Velocity vectors of sand movement during vertical pipe displacement of a) 0.25D b) 0.5D and horizontal pipe displacement of c) 0.25D and e) 0.5D.

Figure 10. Final deformed shape of soil and mesh using MMALE and CEL methods, element concentration is clearly observed in MMALE.

Figure 11. Final velocity vectors of sand movement after enforcement of both vertical and horizontal pipe displacement.

Figure 12. Pipeline response during penetration and lateral displacement.
4 NUMERICAL EXAMPLE 2: SAND COLUMN COLLAPSE

4.1 Background

Collapse of sand column has been extensively studied as an experimental test and benchmark or numerical methods verification. Conventionally, a sand specimen is deposited inside a container. As shown in Figure 13, at least a side of the container is released abruptly which allows the sand to flow on the surface. Then, the corresponding parameters such as run-out distance, slope angle, etc. are studied; see (Lube et al. 2005) for further information.

4.2 Numerical model

(Lube et al. 2005) carried out several experiments on two dimensional sand columns. The results of this experiment are used as a benchmark case for numerical assessment of MMALE and CEL. The evaluation parameters are the run-out distance and sand column height.

The configuration of the numerical model which is obtained from the experiment is shown in Figure 14. The column of sand is at rest until one side of the container is removed to let the soil flow by its own weight. The initial width and height of the soil column is \( d_i = 0.0905 \) m and \( h_i = 0.635 \) m leading to height to width aspect ratio \( a = 7 \). In the experiment, the depth of the soil in direction normal to flow is 0.2 m. It was reported that in this direction no relative difference in run-out distance was observed. Therefore, it is possible to model the experiment in two dimensions. However, due to lack of 2D Eulerian elements in the commercial hydrocodes used, a 3D model with depth of 1 m was developed consisting of hexahedral elements with 1-point integration. The Mohr-Coulomb material model is used in the present study, and the surface friction angle is assumed equal to the internal friction angle of the sand. Material properties are summarized in Table 2 based on a research by (Solowski & Sloan 2013). It should be noticed that the density was assumed by the authors as an average value of sand density (Table 2). In reasonable range of sand density, the effect of this parameter was observed to be negligible.

For both MMALE and CEL, a void region should be defined to allow the soil to flow in this region after the collapse has started. The gravity acceleration is taken as 9.806 m/s\(^2\). The total calculation time of the problem is 2 seconds. Rigid parts are employed to model the container and the flowing surface. The container is assumed smooth and frictionless. The gate is released after in-situ stresses are initialized.

4.3 Results and discussion

Again, the problem was first modeled using the classical Lagrangian approach. In Figure 15, the deformed shape of sand clearly shows the inability of the method for such large material deformations. At about 30% of the calculation time, the mesh quality is significantly reduced. Consequently, the time step size decreased drastically. Even if the termination time was reached, the resulting mesh size would have made the results unreliable due to excessive mesh distortion. In contrast to the Lagrangian method, both CEL and MMALE simulations reached a converged solution. This is due to the implemented advection technique which enables the calculation of sand motion independent of mesh deformation. In Figure 16, the final soil shape is shown. Mesh element size was initially taken as 15 mm for both MMALE and CEL. Despite convergence, the initial CEL model results in a poorly resolved free surface of the collapsing sand column. Hence, the mesh element size was refined to 7.5 mm. The sand column shape after flow was also evaluated in terms of its measured run-out distance and height. This is shown in Figure 17 for different times.

![Figure 13. Schematic view of the sand column problem](image)

<table>
<thead>
<tr>
<th>Table 2 Mohr-Coulomb properties for the sand column collapse model (Solowski &amp; Sloan 2013)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Density (kg/m(^3))</td>
</tr>
<tr>
<td>Friction Angle (°)</td>
</tr>
<tr>
<td>Dilatancy angle (°)</td>
</tr>
<tr>
<td>Cohesion (kPa)</td>
</tr>
<tr>
<td>Poisson ratio</td>
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<tr>
<td>Elastic Modulus (kPa)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3. Calculation time comparison for MMALE and CEL for sand column collapse problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical method</td>
</tr>
<tr>
<td>CEL</td>
</tr>
<tr>
<td>MMALE</td>
</tr>
</tbody>
</table>
Figure 14 Initial configuration of the sand column collapse model

Figure 15. Mesh deformation at an intermediate stage of sand column collapse using a classical Lagrangian method

Figure 16. CEL mesh (above) and MMALE mesh (below) at the end of the calculation

Figure 17. Comparison of the free surface at different time stations for MMALE and CEL with experimental results
Owing to the remeshing feature in MMALE, a mesh density at the free surface comparable to that of the fixed CEL mesh could be reached using a coarser initial mesh. Clearly, the mesh can be adapted to material deformations during remeshing, which renders MMALE computationally less expensive than CEL at comparable accuracy. The computation time for CEL and MMALE using the mesh of Figure 16 are summarized in Table 3.

5 CONCLUSION

In this study, the performance of two numerical analysis approaches tailored for large deformation problems, CEL and MMALE, was evaluated using two example applications. These examples cannot be solved using classical Lagrangian methods since the latter stop at early stages or provide unacceptable results. For the first problem addressing lateral pipeline displacement an analytical solution is available. On the other hand, for the second problem of sand column collapse, experimental measurement is available. Therefore, it was possible to thoroughly investigate both methods and compare their results. Both methods provided comparable results within acceptable calculation time which proves their efficiency and robustness. One of the major differences between MMALE and CEL lies in the remeshing resp. rezoning step. In CEL the mesh is rezoned to its original configuration, while in MMALE the mesh is rezoned to an arbitrary mesh, including the Eulerian (fixed) or Lagrangian mesh as limit cases. The utilized rezoning technique in MMALE has several advantages. At the same mesh size, MMALE interface resolution is generally higher in comparison to CEL. Moreover, an MMALE mesh provides a natural form of solution adaptivity, meaning that mesh density and resolution is increased in areas of interest. On the other hand, it is possible to use coarser meshes in MMALE simulations than in CEL simulations at comparable accuracy in order to reduce calculation times. Additionally, for problems with structural (Lagrangian) parts, MMALE provides more coupling nodes which increases the robustness of the model and decreases the problem of material leakage in CEL methods. The findings of this study highlight that the Multi-Material Arbitrary Lagrangian-Eulerian method is suitable for simulation of large deformations, and it can be considered as a promising tool for modelling more complex geotechnical problems.

REFERENCES